

Comments on the nilpotent constraint of the goldstino superfield

D. M. Ghilencea^{a, b}

^a CERN Theory Division, CH-1211 Geneva 23, Switzerland

^b Theoretical Physics Department, National Institute of Physics and
Nuclear Engineering (IFIN-HH) Bucharest, MG-6 077125, Romania

Abstract

Superfield constraints were often used in the past, in particular to describe the Akulov-Volkov action of the goldstino by a superfield formulation with $L = (\Phi^\dagger \Phi)_D + [(f\Phi)_F + \text{h.c.}]$ endowed with the nilpotent constraint $\Phi^2 = 0$ for the goldstino superfield (Φ) . Inspired by this, such constraint is often used to define the goldstino superfield even *in the presence* of additional superfields, for example in models of “nilpotent inflation”. In this review we show that the nilpotent property is not valid in general, under the assumption of a microscopic (ultraviolet) description of the theory with linear supermultiplets. Sometimes only weaker versions of the nilpotent relation are true such as $\Phi^3 = 0$ or $\Phi^4 = 0$ ($\Phi^2 \neq 0$) in the *infrared* (far below the UV scale) under the further requirement of decoupling all additional scalars (coupling to sgoldstino), something not always possible (e.g. if light scalars exist). In such cases the weaker nilpotent property is not specific to the goldstino superfield anymore. We review the restrictions for the Kahler curvature tensor and superpotential W under which $\Phi^2 = 0$ remains true in infrared, assuming linear supermultiplets in the microscopic description. One can reverse the arguments to demand that the nilpotent condition, initially an infrared property, be extended even in the presence of additional superfields, but this may question the nature of supersymmetry breaking or the existence of a perturbative ultraviolet description with linear supermultiplets.

1 Introduction

Superfield constraints were often used in the past (see [1] for early models) on microscopic (ultraviolet) supersymmetric Lagrangians to project out some of the degrees of freedom (of that superfield) and to obtain in this way non-linear realizations of supersymmetry. For the case of a single superfield, an interaction-free Lagrangian $L = \int d^4\theta \Phi^\dagger \Phi + (\int d^2\theta f\Phi + h.c)$ endowed with the nilpotent constraint $\Phi^2 = 0$ provides [2] a simple superfield description of the famous Akulov-Volkov Lagrangian [3], see also more recent [4]. Here Φ is the goldstino superfield, $\Phi = (\phi, \psi, F)$ where ϕ is the sgoldstino, the scalar superpartner of goldstino ψ . The solution to the nilpotent constraint is $\Phi = \psi\psi/(2F) + \sqrt{2}\theta\psi + \theta\theta F$ which when used in L recovers *onshell* the Akulov-Volkov action [5, 6]. Actually L was itself derived by starting from a general Kahler potential $K(\Phi, \Phi^\dagger)$ and superpotential $W(\Phi)$ after taking the limit of an *infinite* sgoldstino mass [2] giving $\phi = \psi\psi/(2F)$ and thus leading to L . Further, the Akulov-Volkov result was extended to supergravity [7, 8].

The nilpotent property $\Phi^2 = 0$ was then used beyond the Akulov-Volkov action, even in cases when additional superfields are present [4]. Its applications to supersymmetric models were studied together with other projector relations applied to the microscopic Lagrangian, to decouple either bosonic or fermionic superpartners, that we do not discuss here. More interestingly, it was noticed in [4] and verified in general models [9] that the goldstino superfield is the infrared limit of the superconformal symmetry breaking chiral superfield X [10, 11] that breaks the conservation of the Ferrara-Zumino supercurrent [12].

In this review, we start from a general microscopic (UV) Lagrangian with linear supersymmetry *in the presence* of additional superfields $\Phi_i = (\phi_i, \psi_i, F_i)$ and investigate if one can actually have $\Phi^2 = O(1/\Lambda)$ for the goldstino superfield, with Λ the UV scale (related to the Kahler curvature tensor). This would give $\Phi^2 = 0$ *in the infrared* i.e. at vanishing momenta and scales far below Λ . We show that the answer strongly depends on the properties of the Lagrangian. The nilpotent property of the goldstino superfield means that the sgoldstino is decoupled (i.e. it is massive enough to be integrated out via equations of motion). In general this is not always possible since the sgoldstino is often light or even massless at tree level¹ or perturbation theory breaks down. Moreover the sgoldstino is often a mixture of many scalar fields. Only upon the integration of all these scalars (if massive enough) could one hope for a solution of the form $\phi = \psi\psi/(2F)$ and thus for Φ^2 to vanish in the infrared. However, in general $\phi = a_{ij} \psi^i \psi^j + a^{kl} \overline{\psi}_k \overline{\psi}_l + c_{ij}^{kl} \psi^i \psi^j \overline{\psi}_k \overline{\psi}_l$ giving $\Phi^2 \neq 0$, unless additional conditions are met. These issues are often ignored in the literature. The purpose of this work is to review the additional restrictions to be respected by the Kahler potential K and superpotential W in order to have $\Phi^2 = 0$ in the infrared.

One can also reverse the arguments and take a different, easier approach: simply take the

¹as in the O’Raifeartaigh model.

nilpotent constraint as a *definition* for the goldstino superfield even in the presence of additional superfields, without being concerned about the existence of a UV linear description of the goldstino multiplet. One thus gives up the UV microscopic description, which might not even exist (if restrictions like those discussed earlier are not respected). There are various applications of this approach, e.g. [13, 14, 15, 16, 17, 18, 19, 20, 21]. This method is particularly popular in some models of “nilpotent inflation” because in this case one does not need to stabilize the sgoldstino direction since it is a bilinear of fermions, leading to a simplification of problems like moduli stabilization. Note however that the nilpotency of goldstino was initially an infrared property [4, 9], valid at vanishing momenta and scales far below Λ (or the scale of inflation, etc). Questions also remain on the exact nature of supersymmetry breaking and on the existence of a perturbative ultraviolet completion with linear supermultiplets.

Here we adopt the view of a microscopic Lagrangian with linear supermultiplets as the starting point. We review simple counter-examples to the condition $\Phi^2 = 0$ in infrared that show that even in the minimal case of two superfields such constraint is not respected and cannot be used to define the goldstino superfield. For simple K and W weaker versions of this property are often true, such as $\Phi^3 = 0$ or $\Phi^4 = 0$ with $\Phi^2 \neq 0$ and only *after decoupling both scalars* of the theory. Moreover this weaker property applies to both superfields i.e. it is not specific to the goldstino superfield. The reason for the weaker nilpotent relation is simple: for large enough powers, Φ^n vanishes due to the presence of a large power of Grassmann variables; this happens if both scalars of the two superfields are massive enough (to be integrated out) to become combinations of Weyl bilinears ($\phi = a_{ij}\psi^i\psi^j + a^{kl}\bar{\psi}_k\bar{\psi}_l + c_{ij}^{kl}\psi^i\psi^j\bar{\psi}_k\bar{\psi}_l$). The case of a light (matter) scalar in the model can invalidate even these weaker constraints. Assuming linear supermultiplets in a microscopic K and W , the infrared nilpotent property of the goldstino superfield is maintained under restrictive conditions for the Kahler curvature tensor and W that we shall identify. Otherwise $\Phi^2 \neq 0$ in the infrared.

For general K and W , identifying the goldstino superfield can be difficult. One has to identify the ground state and the sgoldstino can be a complicated function of the other scalars. This issue can be avoided in applications since the superconformal symmetry breaking chiral superfield X goes in the infrared to the goldstino superfield $X \rightarrow (8f/3)\Phi$, see [4, 9]. Here \sqrt{f} is the supersymmetry breaking scale and X is the solution to the equation $\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_{\alpha}X$ with $X = (\phi_X, \psi_X, F_X)$; \mathcal{J} is the Ferrara-Zumino multiplet of currents [12]. Further, for given K and W one has [10, 11]

$$X = 4W(\Phi^j) - \frac{1}{3}\bar{D}^2K(\Phi^j, \Phi_j^{\dagger}) - \frac{1}{2}\bar{D}^2Y^{\dagger}(\Phi_j^{\dagger}) \quad (1)$$

where \bar{D}^2Y^{\dagger} is an improvement term. This expression can be used in applications to identify the sgoldstino in the infrared and also to see if $\Phi^2 = \mathcal{O}(1/\Lambda)$ after integrating the scalars. For the examples considered we verify that in the infrared X goes to the goldstino superfield.

2 Nilpotent goldstino superfield: some (counter)examples

2.1 A simple model

Let us first review a simple model [4]

$$K = \Phi^\dagger \Phi - \frac{c}{\Lambda^2} \Phi^2 \Phi^{\dagger 2} - \frac{\tilde{c}}{\Lambda^2} (\Phi^3 \Phi^\dagger + \Phi \Phi^{\dagger 3}) + \mathcal{O}(1/\Lambda^3), \quad W = f \Phi \quad (2)$$

Supersymmetry is broken by non-zero $\langle F \rangle$, so $\Phi = (\phi, \psi, F)$ is a goldstino superfield. The higher dimensional D-terms provide a mass term for the sgoldstino ϕ , while the goldstino is massless, as expected. The scalar potential is

$$V = f^2 [1 + 4c/\Lambda^2 \phi^\dagger \phi + 3\tilde{c}/\Lambda^2 (\phi^2 + \text{h.c.}) + \mathcal{O}(1/\Lambda^3)] \quad (3)$$

The masses of the two real scalars $\varphi_{1,2}$ of ϕ are: $m_{1,2}^2 = f^2 (4c \pm 6\tilde{c})/\Lambda^2$. Thus one must choose $|\tilde{c}| < (2/3)c$ to ensure positive scalar (masses)². Since sgoldstino is massive and goldstino is massless, one can integrate out the former by using its equation of motion at zero momentum and then expanding it about the ground state, to find

$$\phi = -\frac{\psi\psi}{2f} + \mathcal{O}(1/\Lambda) \quad (4)$$

where $\mathcal{O}(1/\Lambda)$ denotes terms suppressed by Λ , e.g. \sqrt{f}/Λ , etc. So the sgoldstino as a composite of goldstini. Note that the limit of restoring supersymmetry $f \rightarrow 0$ in eq.(4) does not exist. Further, with ϕ as in eq.(4), one has that

$$\Phi^2 = \mathcal{O}(1/\Lambda) \quad (5)$$

i.e. its square is vanishing in the infrared limit (defined here by zero momentum and Λ much larger than all other scales in the theory). This limit should be taken with care since it is not always well defined perturbatively. Indeed, one has $m_{1,2}^2 \sim f$ (from supersymmetry breaking) which together with $m_{1,2}^2 < \Lambda^2$ give $\Lambda^2/\rho \sim f < \Lambda^2/\sqrt{\rho}$, for $\rho = 4c \pm 6\tilde{c} > 0$. This range for f is very small and if $\mathcal{O}(\sqrt{f}/\Lambda)$ cannot be neglected then Φ^2 is not vanishing.

From eq.(1) after using the equation of motion $\overline{D}^2 \Phi^\dagger = 4f + \mathcal{O}(1/\Lambda^2)$, one has (ignoring the improvement term)

$$X = 4W - \frac{4}{3}f\Phi + \mathcal{O}(1/\Lambda^2) = \frac{8}{3}f\Phi + \mathcal{O}(1/\Lambda) \quad (6)$$

This verifies that in the infrared and onshell $X \rightarrow (8/3)f\Phi$.

2.2 O’Raifeartaigh model and nilpotent goldstino superfield

Let us now consider a model that includes other fields in addition to the goldstino superfield, such as the O’Raifeartaigh model

$$K = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3, \quad W = \frac{1}{2} h \Phi_1 \Phi_2^2 + m_s \Phi_2 \Phi_3 + f \Phi_1 \quad (7)$$

We assume that $\Phi_{2,3}$ have a large supersymmetric mass (m_s). Φ_1 is the goldstino superfield. Its scalar and fermionic components are both massless at the tree level. At the quantum level the sgoldstino acquires a small mass.

To see this one computes the one-loop correction to the Kahler potential

$$K_{1\text{-loop}} = -\frac{1}{32\pi^2} \text{tr} \left\{ \mathcal{M}^\dagger \mathcal{M} \ln \frac{\mathcal{M}^\dagger \mathcal{M}}{\Lambda^2} \right\} \quad (8)$$

where $\mathcal{M}_{ij} = M_{ij} + \Phi_1 N_{ij}$ is read from re-writing the above superpotential in the form $W = f \Phi_1 + (1/2)(M_{ij} + \Phi_1 N_{ij})\Phi_i \Phi_j$, $i, j = 1, 2, 3$. After integrating out the two massive superfields $\Phi_{2,3}$, the result is a new K_{eff} and W_{eff} shown below

$$K_{\text{eff}} = \Phi_1^\dagger \Phi_1 - \epsilon (\Phi_1^\dagger \Phi_1)^2 + \mathcal{O}(\epsilon^2), \quad W_{\text{eff}} = f \Phi_1, \quad \text{with} \quad \epsilon = \frac{1}{12} \left(\frac{h^2}{4\pi} \right)^2 \frac{1}{|m_s|^2} \quad (9)$$

This result is valid under a simplifying assumption of small supersymmetry breaking: $f h < |m_s|^2$ (for details see [22, 23, 24, 25]). As a result, the mass of the sgoldstino is small but non-zero, as shown by the higher dimensional D-term above generated by quantum corrections: $m_1^2 = 4\epsilon f^2$. However, for a reliable effective theory approach, this mass should be of the order of supersymmetry breaking scale $\mathcal{O}(f)$. Therefore one should have $h^2 \sim \mathcal{O}(4\pi)$ i.e. a nearly *strongly coupled* regime. This indicates that in general it is difficult to generate perturbatively a non-zero mass for sgoldstino and this is expected to be rather light, so integrating it out can be problematic.

As seen in the previous example of eq.(2) with replacements $\tilde{c} = 0$ and $c/\Lambda^2 \rightarrow \epsilon$, we find that $\phi_1 = \psi_1 \psi_1 / (-2f)$. As a result, on the ground state the goldstino superfield satisfies again $\Phi_1^2 = 0$. As before, one can check that $X = 8/3 f \Phi_1 + \mathcal{O}(\epsilon)$ and $X^2 = \mathcal{O}(\epsilon)$, that vanishes in the infrared.

2.3 Akulov-Volkov action from nilpotent goldstino superfield

From the previous examples one may infer the infrared nilpotent property of the goldstino superfield could be more general, then one should be able to relate it to the non-linear realization

of Akulov-Volkov action of the goldstino [3]. Consider then an interaction-free Lagrangian endowed with this nilpotent property

$$\mathcal{L} = \int d^4\theta \Phi^\dagger \Phi + \left\{ \int d^2\theta f \Phi + \text{h.c.} \right\} \quad \text{with} \quad \Phi^2 = 0. \quad (10)$$

The solution to this constraint is

$$\Phi = \gamma \frac{\psi\psi}{F} + \sqrt{2}\theta\psi + \theta\theta F \quad \gamma = 1/2. \quad (11)$$

This is used back into eq.(10) to find the equation of motion for the auxiliary field

$$F = -f - \gamma^2 \frac{\overline{\psi\psi}}{F^{\dagger 2}} \square \left[\frac{\psi\psi}{F} \right] \quad (12)$$

with solution

$$F = -f \left[1 - \frac{\gamma^2}{f^4} \overline{\psi\psi} \square(\psi\psi) - \frac{\gamma^4}{f^8} (\psi\psi)(\overline{\psi\psi}) \square(\psi\psi) \square(\overline{\psi\psi}) \right] \quad (13)$$

Then the onshell Lagrangian is

$$\mathcal{L} = -f^2 + \frac{i}{2} (\psi\sigma^\mu \partial_\mu \overline{\psi} - \partial_\mu \psi \sigma^\mu \overline{\psi}) - \frac{\gamma^2}{f^2} \overline{\psi\psi} \square(\psi\psi) - \frac{\gamma^4}{f^6} (\psi\psi)(\overline{\psi\psi}) \square(\psi\psi) \square(\overline{\psi\psi}) \quad (14)$$

For $\gamma = 1/2$ a non-linear field redefinition shows [5, 6] that eq.(14) is equivalent to the Akulov-Volkov action. The property $\Phi^2 = 0$ is exact. Also, using eq.(1) and the equation of motion for Φ , one has $X = (8/3)f\Phi$, so $X^2 = 0$ too, and there are no other superfields present.

One can also ask if the Akulov-Volkov Lagrangian can be recovered by using instead a weaker constraint $\Phi^n = 0$, $n > 2$, $\Phi^2 \neq 0$. This has a solution² with $\phi = \gamma\psi\psi/F$ with $\gamma \neq 1/2$. However, the limit of an infinite mass of the sgoldstino for any $K(\Phi, \Phi^\dagger)$, $W(\Phi)$ fixes $\gamma = 1/2$ [2]. So the superfield description with $\gamma = 1/2$ in eqs.(10), (14) is unique³.

2.4 A counter-example to $\Phi^2 = 0$

The question is how general the previous examples are when additional fields and interactions are present. Is the goldstino superfield nilpotent when more superfields are present with more

² $\Phi^n = \phi^n + n\sqrt{2}\theta\psi\phi^{n-1} + n\phi^{n-2}[\phi F - (n-1)\psi\psi/2]$ which vanishes for any $\phi \propto \psi\psi/F$, $n > 2$.

³This view is also supported by the fact that there seems to be no mapping [5, 6] of eq.(14) with $\gamma \neq 1/2$ to the Akulov-Volkov Lagrangian. The author thanks S. J. Tylor and S. Kuzenko for this clarification.

complicated K and W ? The examples in Sections 2.1, 2.2 seem to suggest this is indeed the case [4]. Consider however the following example [27]

$$\begin{aligned} K &= \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - \epsilon_1 (\Phi_1^\dagger \Phi_1)^2 - \epsilon_2 (\Phi_2^\dagger \Phi_2)^2 - \epsilon_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &- \epsilon_4 [(\Phi_1^\dagger)^2 \Phi_2^2 + \text{h.c.}] + \mathcal{O}(1/\Lambda^3) \end{aligned} \quad (15)$$

and the superpotential

$$W = f \Phi_1, \quad \epsilon_i = \mathcal{O}(1/\Lambda^2) \quad (16)$$

The scalar potential is

$$V = W_i (K^{-1})^i_j W^j = f^2 (1 + \epsilon_3 |\phi_2|^2 + 4 \epsilon_1 |\phi_1|^2) + \mathcal{O}(1/\Lambda^3) \quad (17)$$

The ground state is $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$ and the scalars masses are: $m_{\phi_1}^2 = 4\epsilon_1 f^2$, $m_{\phi_2}^2 = \epsilon_3 f^2$. Φ_1 is the goldstino superfield. With $\langle F_1 \rangle = -f + \mathcal{O}(\epsilon_i)$, $\langle F_2 \rangle = \mathcal{O}(\epsilon_i)$, after using the equations of motion to integrate out massive $\phi_{1,2}$, one finds that

$$\begin{aligned} \phi_1 &= -\frac{\psi_1 \psi_1}{2f} - \frac{\epsilon_4}{\epsilon_1} \frac{\psi_2 \psi_2}{2f} + \mathcal{O}(1/\Lambda) \\ \phi_2 &= -\frac{\psi_1 \psi_2}{f} + \mathcal{O}(1/\Lambda) \end{aligned} \quad (18)$$

where ϕ_1 is the sgoldstino and ϕ_2 is a matter scalar. This gives that onshell

$$(\Phi_1)^2 = \frac{\epsilon_4}{\epsilon_1} \frac{\psi_2 \psi_2}{(-f)} \left[\frac{\psi_1 \psi_1}{2(-f)} + \sqrt{2} \theta \psi_1 + \theta \theta (-f) \right] + \mathcal{O}(1/\Lambda) \neq 0. \quad (19)$$

The goldstino superfield does not respect the relation $(\Phi_1)^2 = 0$ in infrared⁴. This result does not depend on the UV scale Λ , since in ϵ_4/ϵ_1 this scale cancels out. Could the nilpotent property still be respected? This would require $\epsilon_4 = 0$, which could be respected by demanding for example an additional R-symmetry. Next, the denominator in ϕ_1 and $(\Phi_1)^2$ is related to the sgoldstino mass $m_{\phi_1} = 4\epsilon_1 f^2$ which, if very large, could formally restore the nilpotent property. But this would require $\epsilon_4 \ll \epsilon_1$, which impacts on the convergence of series expansion of Kahler potential (usually $\epsilon_{1,4} \sim 1/\Lambda^2$). Thus, if the goldstino superfield interacts with other superfields, one cannot have $(\Phi_1)^2 = 0$ in infrared without further assumptions.

⁴ Actually this conclusion and similar relations to those in the text also apply off-shell [26, 27].

Note however that a weaker condition is instead respected in the infrared, onshell and also offshell supersymmetry

$$(\Phi_1)^3 = (\Phi_1)^2 \Phi_2 = \Phi_1 (\Phi_2)^2 = (\Phi_2)^3 = 0 \quad (20)$$

These relations are simple consequences of the properties of the two-dimensional Grassmann variables (e.g. $\psi^3 = 0$, $\theta^\alpha \theta^\beta \theta^\gamma = 0$, etc) and are independent of the fields masses. Moreover, note that these relations are symmetric in Φ_1 and Φ_2 , so this weaker nilpotent property e.g. $\Phi_i^3 = 0$ is not specific to the goldstino superfield.

What about the superfield X , eq.(1)? From the equations of motion $\overline{D}^2 \Phi_1^\dagger = 4f + \mathcal{O}(\epsilon_i)$, $\overline{D}^2 \Phi_2^\dagger = \mathcal{O}(\epsilon_i)$, then onshell

$$X = 4f\Phi_1 - \frac{1}{3} \left[\Phi_1 \overline{D}^2 \Phi_1^\dagger + \Phi_2 \overline{D}^2 \Phi_2^\dagger \right] + \mathcal{O}(\epsilon_i) = \frac{8}{3} f \Phi_1 + \mathcal{O}(\epsilon_i) \quad (21)$$

so in the infrared and onshell $X = (8/3)f\Phi_1$ and $X^3 = 0$ but $X^2 \neq 0$.

2.5 Another counter-example to $\Phi^2 = 0$

Consider another example [26]

$$K = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - \epsilon_1 (\Phi_1^\dagger \Phi_1)^2 - \epsilon_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \mathcal{O}(1/\Lambda^3) \quad (22)$$

with

$$W = f\Phi_1 + \frac{\lambda}{3}(\Phi_2)^3, \quad \epsilon_i = \mathcal{O}(1/\Lambda^2) \quad (23)$$

Φ_2 is a matter superfield that now has Yukawa couplings and Φ_1 is the goldstino superfield. Both $\phi_{1,2}$ have masses similar to those in previous section. Integrating them out gives

$$\begin{aligned} \Phi_1 &= a_{ij} \psi_i \psi_j + b_{ij} \overline{\psi}_i \overline{\psi}_j + \sqrt{2}\theta \psi_1 + \theta\theta F_1 \\ \Phi_2 &= c_{ij} \psi_i \psi_j + d_{ij} \overline{\psi}_i \overline{\psi}_j + \sqrt{2}\theta \psi_2 + \theta\theta F_2 \end{aligned} \quad (24)$$

where a summation is understood over $i, j = 1, 2$ while the coefficients $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ are not presented here (offshell they depend on the auxiliary fields). Using the properties of Grassmann variables, one shows that this time an even weaker nilpotent property exists in the infrared (onshell and offshell)

$$(\Phi_1)^4 = (\Phi_1)^3 \Phi_2 = (\Phi_1)^2 (\Phi_2)^2 = \Phi_1 (\Phi_2)^3 = (\Phi_2)^4 = 0, \quad (\Phi_1)^2 \neq 0, \quad \Phi_1 \Phi_2 \neq 0. \quad (25)$$

When $\lambda = 0$, one recovers eq.(20).

For the superfield X , by using the equations of motion of $\Phi_{1,2}$: $(-1/4)\overline{D}^2\Phi_1^\dagger + f = \mathcal{O}(\epsilon_i)$ and $(-1/4)\overline{D}^2\Phi_2^\dagger + \lambda\Phi_2^2 = \mathcal{O}(\epsilon_i)$, one finds from eq.(1)

$$X = 4W - \frac{1}{3} [4f\Phi_1 + 4\lambda\Phi_2^3] + \mathcal{O}(\epsilon_i) = \frac{8}{3}f\Phi_1 + \mathcal{O}(\epsilon_i) \quad (26)$$

This verifies again $X = 8/3 f \Phi_1$ in infrared and onshell, so in this limit $X^4=0$ but $X^2 \neq 0$.

We see that for simple examples with two superfields present $\Phi_1^2 = 0$ is not true in infrared (even after integrating out all scalars). While weaker versions of the nilpotent relation are valid $\Phi_k^n = 0$, $n = 3, 4, \dots$, $k = 1, 2$ after integrating out the scalars, this property is not specific to the goldstino superfield anymore. Moreover, if there are light scalars (that cannot be integrated out) even this weaker version of the nilpotent relation can fail.

2.6 Nilpotent property in a general case: conditions for use

From these counter-examples it is clear that the nilpotent property of the goldstino superfield is not valid in general in the presence of more superfields and under the assumption of an initial microscopic (UV) Lagrangian with linear supermultiplets. Here we review the situation for more general K and W and consider the following case [9]⁵

$$\begin{aligned} L &= \int d^4\theta K(\Phi^i, \Phi_j^\dagger) + \left\{ \int d^2\theta W(\Phi^i) + \text{h.c.} \right\} \\ &= K_i^j \left[\partial_\mu \phi^i \partial^\mu \phi_j^\dagger + \frac{i}{2} (\psi^i \sigma^\mu \mathcal{D}_\mu \bar{\psi}_j - \mathcal{D}_\mu \psi^i \sigma^\mu \bar{\psi}_j) \right] - W^k (K^{-1})_k^i W_i \\ &\quad - \frac{1}{2} \left[(W_{ij} - \Gamma_{ij}^m W_m) \psi^i \psi^j + \text{h.c.} \right] + \frac{1}{4} R_{ij}^{kl} \psi^i \psi^j \bar{\psi}_k \bar{\psi}_l, \quad \text{with} \quad R_{ij}^{kl} \equiv K_{ij}^{kl} - K_{ij}^n \Gamma_n^{kl} \end{aligned} \quad (27)$$

in a standard notation⁶. L is derived from the offshell form \mathcal{L}^7 . Denote

$$f_i = W_i(\langle \phi^m \rangle), \quad f_{ij} = W_{ij}(\langle \phi^m \rangle), \quad f_{ijk} = W_{ijk}(\langle \phi^m \rangle), \quad f^i = W^i(\langle \phi^m \rangle), \text{ etc.} \quad (28)$$

In normal coordinates that we use in the following⁸ the curvature tensor is $R_{ij}^{lm} = k_{ij}^{lm}$ where

⁵Hereafter we use superscripts to label fields $\Phi^i = (\phi^i, \psi^i, F^i)$ not to be confused with powers (Φ_i^\dagger) for h.c.)

⁶ We use $K_i \equiv \partial K / \partial \phi^i$, $K^n \equiv \partial K / \partial \phi_n^\dagger$, $K_i^n \equiv \partial^2 K / (\partial \phi^i \partial \phi_n^\dagger)$, $W_j = \partial W / \partial \phi^j$, $W^j = (W_j)^\dagger$, in which $W = W(\phi^i)$, $K = K(\phi^i, \phi_j^\dagger)$ are now functions of scalars. Note $K_{ij}^k \sim 1/\Lambda$, $K_{km}^{ij} \sim 1/\Lambda^2$, with Λ the UV cutoff. Also $(\Gamma_{jk}^l)^\dagger = \Gamma_l^{jk}$, $(K_{jk}^m)^\dagger = K_m^{jk}$, $(K^{-1})_m^l = (K^{-1})^l_m$, $\mathcal{D}_\mu \psi^l \equiv \partial_\mu \psi^l - \Gamma_{jk}^l (\partial_\mu \phi^j) \psi^k$, $\Gamma_{jk}^l = (K^{-1})_m^l K_{jk}^m$, $F_m^\dagger = -(K^{-1})_m^i W_i + (1/2) \Gamma_m^{lj} \bar{\psi}_l \bar{\psi}_j$. In complex geometry R_{ij}^{kl} denotes $(R_{ki}^j)_{\bar{l}}^l = K_{ij}^{k\bar{l}} - K_{ij\bar{p}} K^{\bar{p}n} K_{n\bar{k}l}$.

⁷ $\mathcal{L} = K_i^j [\partial_\mu \phi^i \partial^\mu \phi_j^\dagger + \frac{i}{2} (\psi^i \sigma^\mu \mathcal{D}_\mu \bar{\psi}_j - \mathcal{D}_\mu \psi^i \sigma^\mu \bar{\psi}_j) + F^i F_j^\dagger] + \frac{1}{4} K_{ij}^{kl} \psi^i \psi^j \bar{\psi}_k \bar{\psi}_l + [(W_k - \frac{1}{2} K_k^{ij} \bar{\psi}_i \bar{\psi}_j) F^k - \frac{1}{2} W_{ij} \psi^i \psi^j + \text{h.c.}]$

⁸In normal coordinates $k_j^i = \delta_j^i$, $k_{jk\dots}^i = k_i^{jk\dots} = 0$, $R_{ij}^{kl} = k_{ij}^{kl}$ where $k_{j\dots}^i$ are the values of $K_{j\dots}^i$ computed on the ground state $\langle \phi^k \rangle$, $\langle F^k \rangle$, $\langle \psi^k \rangle = 0$ with the field fluctuations given by $\delta \phi^i = \phi^i - \langle \phi^i \rangle$.

k_{ij}^{lm} are the values of K_{ij}^{lm} computed on the ground state. Further, from the eqs of motion for F^i , ϕ^i one has $k_i^j \langle F_j^\dagger \rangle + f_i = 0$ and $k_{im}^j \langle F^i \rangle \langle F_j^\dagger \rangle + f_{km} \langle F^m \rangle = 0$ giving $\langle F_j^\dagger \rangle = -f_j$, and $f_{km} \langle F^m \rangle = 0$. SUSY breaking requires at least one non-vanishing $\langle F_j \rangle$, so $\det f_{ij} = 0$. The fermions mass matrix $W_{ij} - \Gamma_{ij}^m W_m$ equals f_{ij} in normal coordinates. The scalar mass matrix

$$M_S^2 = \begin{bmatrix} \langle V_l^k \rangle & \langle V_{kl} \rangle \\ \langle V^{kl} \rangle & \langle V_k^l \rangle \end{bmatrix} = \begin{bmatrix} f^{ik} f_{il} - k_{il}^{jk} f^i f_j & f_{jkl} f^j \\ f^{jkl} f_j & f^{il} f_{ik} - k_{ik}^{jl} f^i f_j \end{bmatrix}, \quad (29)$$

where $V_l^k = \partial^2 V / (\partial \phi^l \partial \phi_k^\dagger)$, $V_{kl} = \partial^2 V / (\partial \phi^l \partial \phi^k)$. We assume in the following that

$$f_{ij} = 0, \quad f^{ijl} f_l = 0, \quad \text{and} \quad f^{ijlm} = 0, \quad i, j = 1, 2. \quad (30)$$

which simplify the scalar mass matrix and also enable one to integrate ϕ^i in terms of massless fermions without further restriction. For two superfields the mass eigenvalues are

$$m_{\phi^{1,2}}^2 = \frac{1}{2} \left[-k_{mk}^{mj} f^k f_j \pm \sqrt{\Delta} \right], \quad \Delta = (k_{mk}^{mj} f^k f_j)^2 - 4 \det(k_{mk}^{pj} f^k f_j) \quad (31)$$

where \det is over the free indices and $i, j, k, m \dots = 1, 2$. Consider now the transformation

$$\begin{aligned} \tilde{\Phi}^1 &= \frac{1}{f} (f_1 \delta \Phi^1 + f_2 \delta \Phi^2), \\ \tilde{\Phi}^2 &= \frac{1}{f} (-f_2 \delta \Phi^1 + f_1 \delta \Phi^2), \quad f = \sqrt{f_k f^k} \end{aligned} \quad (32)$$

where $\delta \Phi^j = (\delta \phi^j, \psi^j, F^j)$ and $\delta \phi^j \equiv \phi^j - \langle \phi^j \rangle$. \tilde{F}^1 is a combination of auxiliary fields that ‘‘collects’’ supersymmetry breaking from all directions f^j ; this combination of F^j also dictates that of their superfields $\delta \Phi^j$ for spontaneous supersymmetry breaking, hence $\tilde{\Phi}^1$ is the goldstino superfield and $\tilde{\Phi}^2$ is normal to it.

Let us now discuss the decoupling of the scalars and check under what conditions the goldstino superfield is nilpotent in infrared. We integrate the scalars which become combinations of the massless fermions. From eq.(27) the eq of motion of ϕ_l^\dagger at zero-momentum (infrared) becomes, after expanding it about the ground state

$$k_{kj}^{il} \delta \phi^j f^k f_i + \frac{1}{2} k_{ij}^{lm} f_m \psi^i \psi^j - \frac{1}{2} f^{ijl} \bar{\psi}_i \bar{\psi}_j + \mathcal{O}(1/\Lambda^3) = 0, \quad i, j, k \dots = 1, 2. \quad (33)$$

Taking $l = 1, 2$, one solves this system for $\delta \phi^{1,2}$ to find

$$\begin{aligned}
\delta\phi^1 &= \frac{1}{2\det(k_{lm}^{kn} f_n f^m)} \left[A_{ij} \psi^i \psi^j + B^{ij} \bar{\psi}_i \bar{\psi}_j \right] + \mathcal{O}(1/\Lambda) \\
\delta\phi^2 &= \frac{1}{2\det(k_{lm}^{kn} f_n f^m)} \left[C_{ij} \psi^i \psi^j + D^{ij} \bar{\psi}_i \bar{\psi}_j \right] + \mathcal{O}(1/\Lambda)
\end{aligned} \tag{34}$$

with

$$\begin{aligned}
A_{ij} &= (k_{ij}^{2p} k_{2s}^{1r} - k_{ij}^{1p} k_{2s}^{2r}) f^r f_s f_p, & B^{ij} &= -f^{ij2} k_{2s}^{mr} f^s f_m f_r (f_1)^{-1}, \\
C_{ij} &= (k_{ij}^{1p} k_{1s}^{2r} - k_{ij}^{2p} k_{1s}^{1r}) f^r f_s f_p, & D^{ij} &= -f^{ij1} k_{1s}^{mr} f^s f_m f_r (f_2)^{-1}.
\end{aligned} \tag{35}$$

The fields $\delta\phi^{1,2}$ are suppressed by the product of sgoldstino and matter scalar masses.

Imposing the nilpotent property $(\tilde{\Phi}_1)^2 = \mathcal{O}(1/\Lambda)$, one finds from eqs.(32), (34)

$$\begin{aligned}
f_p f_r f^s [k_{ij}^{2p} (f_1 k_{2s}^{1r} - f_2 k_{1s}^{1r}) - k_{ij}^{1p} (f_1 k_{2s}^{2r} - f_2 k_{1s}^{2r})] + \det(k_{ns}^{mr} f_r f^s) \frac{f_i f_j}{f_k f^k} &= 0 \\
k_{ls}^{mr} f^{ijl} f^s f_m f_r &= 0,
\end{aligned} \tag{36}$$

where $i, j = 1, 2$ are fixed. If these conditions are respected, then the goldstino superfield satisfies $(\tilde{\Phi}^1)^2 = 0$ in infrared, with finite scalar masses. This is true in the presence of superpotential interactions, with more sources of supersymmetry breaking, after starting from a (UV) microscopic Lagrangian with linear superfields. Eqs.(36) bring constraints on the ultraviolet region controlled by the curvature tensor $R_{ij}^{lm} = k_{ij}^{lm}$.

To illustrate further the above result, consider the simpler case of only one field breaking supersymmetry and take $f_2 = 0$, $f_1 \neq 0$. Eqs.(34) simplify

$$\begin{aligned}
\delta\phi^1 &= -\frac{\psi^1 \psi^1}{2 f^1} + \frac{\det(k_{2j}^{1i})}{\det(k_{1n}^{1m})} \frac{\psi^2 \psi^2}{2 f^1} - \frac{k_{21}^{11} f^{ij2}}{\det(k_{1n}^{1m})} \frac{\bar{\psi}_i \bar{\psi}_j}{2 |f_1|^2} + \mathcal{O}(1/\Lambda) \\
\delta\phi^2 &= -\frac{\psi^1 \psi^2}{f^1} + \frac{k_{11}^{12} k_{22}^{11} - k_{11}^{11} k_{22}^{12}}{\det(k_{1n}^{1m})} \frac{\psi^2 \psi^2}{2 f^1} + \frac{k_{11}^{11} f^{ij2}}{\det(k_{1n}^{1m})} \frac{\bar{\psi}_i \bar{\psi}_j}{2 |f_1|^2} + \mathcal{O}(1/\Lambda)
\end{aligned} \tag{37}$$

which generalise the results in Sections 2.4, 2.5, such as eq.(18). The terms proportional to f^{ij2} are dominant since they grow like Λ^2 , ($k_{lm}^{ij} \sim 1/\Lambda^2$). The coefficients of $\psi^i \psi^j$ are independent of Λ but still depend on the couplings in the microscopic Lagrangian. Eqs.(36) also simplify to

$$\det(k_{2j}^{1i}) = 0, \quad \text{and} \quad f^{ij2} k_{12}^{11} = 0. \tag{38}$$

Then $(\tilde{\Phi}_1)^2 = \mathcal{O}(1/\Lambda)$ in the presence of trilinear interactions (as also seen from eq.(37))⁹.

As a further illustration, consider $W = f_1 \Phi^1 + \lambda/3! (\Phi_2)^3$ with a nearly massless matter scalar, which demands $\det(k_{1m}^{1n}) \approx 0$ giving $m_{\tilde{\phi}_1}^2 \approx -(k_{11}^{11} + k_{12}^{12})f_1^2$, $m_{\tilde{\phi}_2}^2 \approx 0$. One can re-do the above calculation and integrate the sgoldstino only to find

$$\delta\phi^1 = -\frac{1}{f_1 k_{11}^{11}} [(1/2) k_{mn}^{11} \psi^m \psi^n + f_1 \delta\phi^2 k_{12}^{11}] + \mathcal{O}(1/\Lambda) \quad (39)$$

Then no power of the goldstino superfield can vanish in infrared, unless $k_{12}^{11} = k_{22}^{11} = 0$.

What about superconformal X ? Consider now that $W = f_i \Phi^i + (1/3!) f_{ijk} \Phi^i \Phi^j \Phi^k$ and the equation of motion of Φ^i : $-1/4 \bar{D}^2 \Phi_i^\dagger + f_i + 1/2 f_{ijk} \Phi^j \Phi^k + \mathcal{O}(1/\Lambda) = 0$ where $\mathcal{O}(1/\Lambda)$ accounts for higher dimensional Kahler terms. Then from eq.(1) one finds, up to improvement terms: $X = 4W - (1/3) \bar{D}^2 K = 8/3 f_i \Phi^i + \mathcal{O}(1/\Lambda) = (8/3) f \tilde{\Phi}^1$ after using $\bar{D}^2 K = \Phi^i \bar{D}^2 \Phi_i^\dagger + \mathcal{O}(1/\Lambda)$ and eq.(32). This result is valid in the infrared, at scales/momenta far below Λ and verifies that X flows into goldstino superfield $\tilde{\Phi}^1$. Then the same restrictions regarding the validity of the nilpotent constraint for $\tilde{\Phi}^1$ apply to X too.

2.7 Avoiding the nilpotent constraint

The use of the nilpotent constraint to define the goldstino superfield in the presence of additional superfields can be avoided in applications. One simply uses eq.(1) which recovers in infrared the goldstino superfield, so that offshell^{10,11}

$$\tilde{\Phi}^1 = \frac{3}{8f} \left[4W(\Phi^i) + \frac{4}{3} \Phi^i (F_i^\dagger + i\sqrt{2}\theta \partial_\mu \sigma^\mu \bar{\psi}_i - \theta\theta \square \phi_i^\dagger) + \mathcal{O}(1/\Lambda) \right] \quad (40)$$

with $f = \sqrt{f_i f^i}$ and its onshell form

$$\tilde{\Phi}^1 = \frac{3}{8f} \left[4W(\Phi^i) - \frac{4}{3} \Phi^i \frac{\partial W}{\partial \Phi^i} + \mathcal{O}(1/\Lambda) \right] \quad (41)$$

from which the expression of sgoldstino is obvious. This form does not integrate out the other scalar fields¹². Eqs.(40), (41) do not depend on K in leading order and can be used in applications that need the expression of the goldstino superfield and its sgoldstino in infrared.

⁹ If $\tilde{\phi}^1$ of (32) is a mass eigenstate (which happens if $k_{12}^{11} = k_{11}^{12} = 0$) then eq.(38) reduces to $k_{22}^{11} k_{12}^{12} = 0$.

¹⁰ Assuming the expression in eq.(1) is indeed valid offshell.

¹¹ One has $\bar{D}^2 \Phi_i^\dagger = -4(F_i^\dagger, i \partial_\mu \sigma^\mu \bar{\psi}_i, -\square \phi_i^\dagger)$ for $\Phi^i = (\phi^i, \psi^i, F^i)$ and that $K = \Phi_i^\dagger \Phi^i + \mathcal{O}(1/\Lambda)$.

¹² When this is possible, the sgoldstino becomes a combination of Weyl bilinears, as seen earlier.

3 Conclusions

Superfield constraints are often used to project out some degrees of freedom of the microscopic Lagrangian and provide a non-linear realization of supersymmetry. The Akulov-Volkov is the celebrated example, described by a free action with $L = [\Phi^\dagger \Phi]_D + [f\Phi_F + \text{h.c.}]$, with a constraint for goldstino superfield $\Phi^2 = 0$ and solution $\Phi = \psi\psi/(2F) + \sqrt{2}\theta\psi + \theta\theta F$. The constraint projects out the sgoldstino which becomes a bilinear of goldstini. L above was initially derived perturbatively from a general, microscopic Lagrangian for Φ in which one decouples the sgoldstino (by taking its mass to infinity), leading to the above solution for Φ and thus to L .

Inspired by this result, the nilpotent property $\Phi^2 = 0$ is sometimes used in the literature to define the goldstino superfield even *in the presence* of additional superfields $\Phi^i = (\phi^i, \psi^i, F^i)$. This procedure can lead to incorrect results. We first reviewed counter-examples to $\Phi^2 = 0$ in the infrared when starting with microscopic Lagrangians with two linear superfields present. We identified the goldstino superfield and checked the nilpotent property after integrating out *all scalars* of the theory; then the sgoldstino becomes a combination of Weyl bilinears $\phi = a_{ij}\psi^i\psi^j + b^{kl}\bar{\psi}_k\bar{\psi}_l$ and the nilpotent property is not respected. In some cases a weaker version of this property is valid such as $\Phi^3 = 0$ or $\Phi^4 = 0$ with $\Phi^2 \neq 0$. The reason is simple: for large enough powers n , Φ^n vanishes in the infrared, once its scalar component is of the type shown above. Moreover, this weaker property applies to both superfields i.e. it is not specific to the goldstino superfield.

We then reviewed more general cases. Assuming linear supermultiplets and a microscopic Lagrangian with general $K(\Phi^i, \Phi_i^\dagger)$ and $W(\Phi^i)$ and more sources of supersymmetry breaking, we found the conditions for the Kahler curvature tensor and superpotential couplings under which one can still have $\Phi^2 = 0$ in the infrared, after integrating out the scalars. These conditions are actually very restrictive for the model and for the ultraviolet region (controlled by the Kahler curvature tensor). If massless scalars exist (coupled to sgoldstino) even the weaker versions of the nilpotent property can be invalidated.

The use of the nilpotent condition of the goldstino superfield can be avoided. Since the superconformal symmetry breaking chiral superfield X goes in infrared to the goldstino superfield, one can just use its known expression (determined by K , W and only by W up to $\mathcal{O}(1/\Lambda)$) to obtain the (infrared) sgoldstino in terms of the other scalars. This expression can then be used in applications. The scalars can eventually be integrated out, so the sgoldstino becomes a combination of Weyl fermions bilinears, leading to $\Phi^2 \neq 0$ unless the aforementioned conditions are respected. This is the picture if one assumes the initial existence of a perturbative UV description of the theory with linear supermultiplets.

One can also proceed in the opposite way: simply use the nilpotent property $\Phi^2 = 0$ as a definition for the goldstino multiplet even in the presence of additional superfields. Doing so

is popular e.g. in “nilpotent inflation” models in which goldstino superfield is present, since in this case the sgoldstino becomes a bilinear of goldstini and one does not have to stabilize this field direction, simplifying the calculation. A good question to ask is then whether the nilpotent relation which initially was only an *infrared* property (on the ground state) can be extended (to non-zero momenta, at the scale of inflation, etc). Questions can also be asked on the nature of supersymmetry breaking in this case and on the microscopic (UV) description of such models in terms of linear supermultiplets. In fact such UV completion might not exist perturbatively (unless the restrictive conditions mentioned earlier are respected).

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